

Foundations of R Software

Lecture 15

Basics of Calculations

::::

Matrix Operations – Mathematical and Other Operations

Shalabh

Department of Mathematics and Statistics

Indian Institute of Technology Kanpur

Addition and subtraction of matrices

Example:

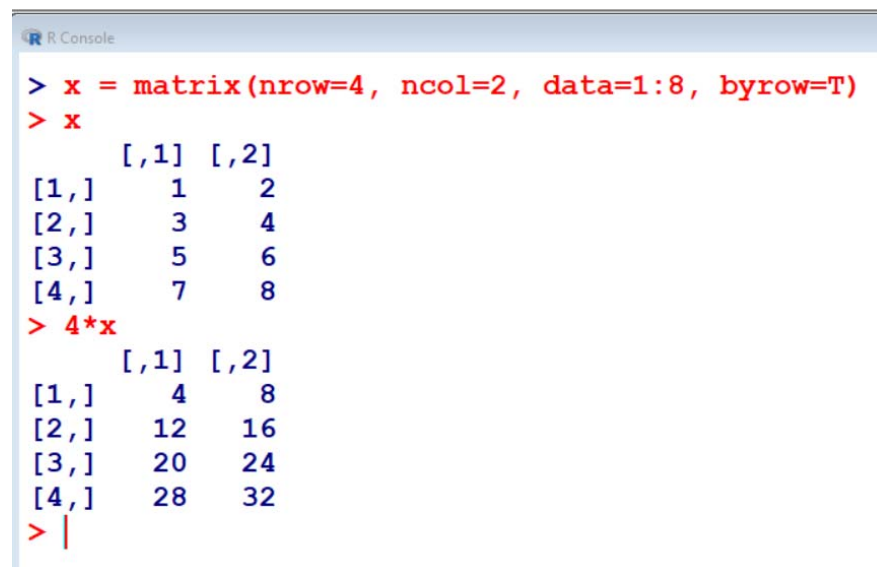
```
> x = matrix(nrow=4, ncol=2, data=1:8, byrow=T)
```

```
> x
```

	[,1]	[,2]
[1,]	1	2
[2,]	3	4
[3,]	5	6
[4,]	7	8

```
> 4*x
```

	[,1]	[,2]
[1,]	4	8
[2,]	12	16
[3,]	20	24
[4,]	28	32



```
R Console
> x = matrix(nrow=4, ncol=2, data=1:8, byrow=T)
> x
      [,1] [,2]
[1,]    1    2
[2,]    3    4
[3,]    5    6
[4,]    7    8
> 4*x
      [,1] [,2]
[1,]     4    8
[2,]    12   16
[3,]    20   24
[4,]    28   32
> |
```

Addition and subtraction of matrices

Example:

```
> x + 4*x
```

	[,1]	[,2]
[1,]	5	10
[2,]	15	20
[3,]	25	30
[4,]	35	40

```
> 4*x - x
```

	[,1]	[,2]
[1,]	3	6
[2,]	9	12
[3,]	15	18
[4,]	21	24

```
R Console
> x
      [,1] [,2]
[1,]    1    2
[2,]    3    4
[3,]    5    6
[4,]    7    8
> 4*x
      [,1] [,2]
[1,]    4    8
[2,]   12   16
[3,]   20   24
[4,]   28   32
> x + 4*x
      [,1] [,2]
[1,]    5   10
[2,]   15   20
[3,]   25   30
[4,]   35   40
> 4*x - x
      [,1] [,2]
[1,]    3    6
[2,]    9   12
[3,]   15   18
[4,]   21   24
> |
```

Multiplication of matrices

Multiplication of matrices can be executed with the operator `%*%`

```
x = matrix(nrow=4, ncol=2, data=1:8, byrow=T)
```

```
y = matrix(nrow=2, ncol=4, data=11:18, byrow=T)
```

```
> x
```

	[, 1]	[, 2]
[1,]	1	2
[2,]	3	4
[3,]	5	6
[4,]	7	8

```
> y
```

	[, 1]	[, 2]	[, 3]	[, 4]
[1,]	11	12	13	14
[2,]	15	16	17	18

Multiplication of matrices

> x%y**

	[, 1]	[, 2]	[, 3]	[, 4]
[1,]	41	44	47	50
[2,]	93	100	107	114
[3,]	145	156	167	178
[4,]	197	212	227	242

> y%x**

	[, 1]	[, 2]
[1,]	210	260
[2,]	274	340

```
R Console
> x = matrix(nrow=4, ncol=2, data=1:8, byrow=T)
>
> y = matrix(nrow=2, ncol=4, data=11:18, byrow=T)
> x
      [,1] [,2]
[1,]    1    2
[2,]    3    4
[3,]    5    6
[4,]    7    8
> y
      [,1] [,2] [,3] [,4]
[1,]   11   12   13   14
[2,]   15   16   17   18
> x**%y
      [,1] [,2] [,3] [,4]
[1,]    41    44    47    50
[2,]    93   100   107   114
[3,]   145   156   167   178
[4,]   197   212   227   242
> y**%x
      [,1] [,2]
[1,]   210   260
[2,]   274   340
> |
```

Multiplication of matrices

Another example: Consider the multiplication of X' and X

```
> x = matrix(nrow=4, ncol=2, data=1:8, byrow=T)
```

```
> x
```

	[, 1]	[, 2]
[1,]	1	2
[2,]	3	4
[3,]	5	6
[4,]	7	8

```
> t(x)
```

	[, 1]	[, 2]	[, 3]	[, 4]
[1,]	1	3	5	7
[2,]	2	4	6	8

Multiplication of matrices

```
> t(x) %*% x
```

	[,1]	[,2]
[1,]	84	100
[2,]	100	120

```
> x %*% t(x)
```

	[,1]	[,2]	[,3]	[,4]
[1,]	5	11	17	23
[2,]	11	25	39	53
[3,]	17	39	61	83
[4,]	23	53	83	113

```
R Console
> x := matrix(nrow=4, ncol=2, data=1:8, byrow=T)
> x
      [,1] [,2]
[1,]    1    2
[2,]    3    4
[3,]    5    6
[4,]    7    8
> t(x)
      [,1] [,2] [,3] [,4]
[1,]    1    3    5    7
[2,]    2    4    6    8
> t(x) %*% x
      [,1] [,2]
[1,]   84  100
[2,]  100  120
> x %*% t(x)
      [,1] [,2] [,3] [,4]
[1,]    5   11   17   23
[2,]   11   25   39   53
[3,]   17   39   61   83
[4,]   23   53   83  113
```

Multiplication of matrices

Cross Product of a matrix X , $X'X$, with `crossprod()`

```
> x = matrix(nrow=4, ncol=2, data=1:8, byrow=T)
```

```
> x
```

	[, 1]	[, 2]
[1,]	1	2
[2,]	3	4
[3,]	5	6
[4,]	7	8

```
> t(x)
```

	[, 1]	[, 2]	[, 3]	[, 4]
[1,]	1	3	5	7
[2,]	2	4	6	8

Multiplication of matrices

```
> crossprod(x)
```

```
      [,1] [,2]  
[1,]    84  100  
[2,]   100  120
```

```
R Console  
  
> x = matrix(nrow=4, ncol=2, data=1:8, byrow=T)  
> x  
      [,1] [,2]  
[1,]    1    2  
[2,]    3    4  
[3,]    5    6  
[4,]    7    8  
> crossprod(x)  
      [,1] [,2]  
[1,]    84  100  
[2,]   100  120  
>  
> t(x) %*% x  
      [,1] [,2]  
[1,]    84  100  
[2,]   100  120  
> |
```

Note: Command `crossprod()` executes the multiplication faster than the conventional method with `t(x)%*%x`

Concatenating matrices

Concatenating matrices row wise: `rbind(x, y)`

Concatenating matrices column wise: `cbind(x, y)`

```
x = matrix(nrow=3, ncol=2, data=1:6, byrow=T)
```

```
y = matrix(nrow=3, ncol=2, data=11:16, byrow=T)
```

<pre>> x</pre>	<pre>> y</pre>	<pre>> rbind(x, y)</pre>
<pre> [,1] [,2]</pre>	<pre> [,1] [,2]</pre>	<pre> [,1] [,2]</pre>
<pre>[1,] 1 2</pre>	<pre>[1,] 11 12</pre>	<pre>[1,] 1 2</pre>
<pre>[2,] 3 4</pre>	<pre>[2,] 13 14</pre>	<pre>[2,] 3 4</pre>
<pre>[3,] 5 6</pre>	<pre>[3,] 15 16</pre>	<pre>[3,] 5 6</pre>
		<pre>[4,] 11 12</pre>
		<pre>[5,] 13 14</pre>
		<pre>[6,] 15 16</pre>

Concatenating matrices

Concatenating matrices row wise: `rbind(x, y)`

Concatenating matrices column wise: `cbind(x, y)`

```
x = matrix(nrow=3, ncol=2, data=1:6, byrow=T)
```

```
y = matrix(nrow=3, ncol=2, data=11:16, byrow=T)
```

<pre>> x</pre>	<pre>> y</pre>
<pre> [,1] [,2]</pre>	<pre> [,1] [,2]</pre>
<pre>[1,] 1 2</pre>	<pre>[1,] 11 12</pre>
<pre>[2,] 3 4</pre>	<pre>[2,] 13 14</pre>
<pre>[3,] 5 6</pre>	<pre>[3,] 15 16</pre>

<pre>> cbind(x, y)</pre>
<pre> [,1] [,2] [,3] [,4]</pre>
<pre>[1,] 1 2 11 12</pre>
<pre>[2,] 3 4 13 14</pre>
<pre>[3,] 5 6 15 16</pre>

Concatenating matrices

Concatenating matrices row wise: `rbind(x, y)`

Concatenating matrices column wise: `cbind(x, y)`

```
R Console
> x = matrix(nrow=3, ncol=2, data=1:6, byrow=T)
> y = matrix(nrow=3, ncol=2, data=11:16, byrow=T)
> x
      [,1] [,2]
[1,]     1     2
[2,]     3     4
[3,]     5     6
> y
      [,1] [,2]
[1,]    11    12
[2,]    13    14
[3,]    15    16
> rbind(x, y)
      [,1] [,2]
[1,]     1     2
[2,]     3     4
[3,]     5     6
[4,]    11    12
[5,]    13    14
[6,]    15    16
> cbind(x, y)
      [,1] [,2] [,3] [,4]
[1,]     1     2    11    12
[2,]     3     4    13    14
[3,]     5     6    15    16
```

Inverse of matrix

`solve()` finds the inverse of a positive definite matrix

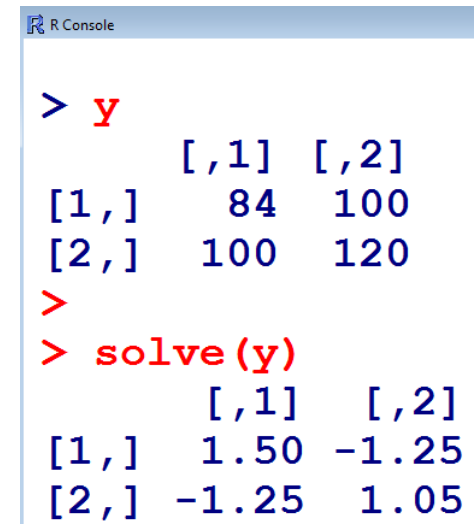
```
> y = matrix( nrow = 2, ncol = 2, byrow = T,  
data = c(84,100,100,120))
```

```
> y
```

```
      [,1] [,2]  
[1,]   84  100  
[2,]  100  120
```

```
> solve(y)
```

```
      [,1] [,2]  
[1,]  1.50 -1.25  
[2,] -1.25  1.05
```



```
R Console  
  
> y  
      [,1] [,2]  
[1,]   84  100  
[2,]  100  120  
>  
> solve(y)  
      [,1] [,2]  
[1,]  1.50 -1.25  
[2,] -1.25  1.05
```

Eigen values and eigen vectors of matrix

eigen() finds the eigen values and eigen vectors of a positive definite matrix

```
y = matrix( nrow = 2, ncol = 2, byrow = T, data  
= c(84,100,100,120) )
```

```
> y
```

```
      [,1] [,2]  
[1,]   84  100  
[2,]  100  120
```

Eigen values and eigen vectors of matrix

```
> eigen(y)
```

```
$values
```

```
[1] 203.6070864    0.3929136
```

```
$vectors
```

```
          [,1]      [,2]  
[1, ] 0.6414230 -0.7671874  
[2, ] 0.7671874  0.6414230
```

```
R Console  
  
> y  
      [,1] [,2]  
[1,]    84  100  
[2,]   100  120  
> eigen(y)  
$values  
[1] 203.6070864    0.3929136  
  
$vectors  
          [,1]      [,2]  
[1,] 0.6414230 -0.7671874  
[2,] 0.7671874  0.6414230
```